

# Localized plasmons in point contacts

Henrik Bruus<sup>†</sup> and Karsten Flensberg<sup>‡</sup>

<sup>†</sup> Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

<sup>‡</sup> Danish Institute of Fundamental Metrology, Anker Engeldunds Vej 1, DK-2800 Lyngby, Denmark

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**Abstract.** Using a hydrodynamic model of the electron fluid in a point contact geometry we show that localized plasmons are likely to exist near the constriction. We attempt to relate these plasmons to the recent experimental observation of deviations of the quantum point contact conductance from ideal integer quantization. As a function of temperature, this deviation exhibits an activated behaviour,  $\exp(-T_a/T)$ , with a density-dependent activation temperature  $T_a$  of the order of 2 K. We suggest that  $T_a$  can be identified with the energy needed to excite localized plasmons, and we discuss the conductance deviations in terms of a simple theoretical model involving quasiparticle lifetime broadening due to coupling to the localized plasmons.

## 1. Introduction

Quantized conduction through a narrow point contact is one of the key effects in mesoscopic physics; the quantum point contact remains an important testing ground for the description of mesoscopic phenomena. Recently, significant deviations from the Landauer–Büttiker theory have been observed in quantum point contacts in the temperature dependence of the conductance quantization [1, 2] and as a so-called ‘0.7’ structure or quasi plateau, appearing around 0.7 times the conductance quantum  $2e^2/h$  [3]. Invoking a Luttinger liquid approach [4], the deviations have been discussed in terms of interaction effects [5–7]. However, firm conclusions have been difficult to obtain partly due to the narrow temperature range (0.1–4 K) in which the effect can be studied in conventional split gate quantum point contacts, where relatively close-lying one-dimensional subbands are formed.

Important progress was provided by the appearance of strongly confined GaAs quantum point contacts using a combination of shallow etching and a top gate [8]. In these new samples the conduction quantization can be followed up to around 30 K. In subsequent work [9] these samples were used to study the temperature dependence of deviations from perfect conductance quantization. At low temperature ( $\sim 0.05$  K) almost ideal quantized conductance is observed for the first conduction plateau, but deviations develop as the temperature is increased. The enlarged temperature range allowed for the observation of activated temperature dependence of these deviations:  $\delta G(T) \propto \exp(-T_a/T)$ . Furthermore, by changing the top gate it was found that  $T_a$  increases with increasing density. An explanation could not be found using the standard single-particle picture, and in the brief theory section of [9] we therefore suggested the inclusion of collective effects through plasmons. In short, we identified  $T_a$  as the energy

needed to excite localized plasmons, and we discussed the conductance in terms of a simple theoretical model involving the additional effect of electrons scattering off the localized plasmons. In the present theoretical work we elaborate on that idea. In a point contact the charge is of course depleted. In order to study the collective excitations of such a system, we can approach it from two limits: (i) squeezing a homogeneous electron liquid, or (ii) connecting two spatially separated liquids. Below we follow the first approach, and we argue from a hydrodynamic model that localized plasmons may exist in realistic situations.

## 2. Plasmons of a homogeneous electron liquid in a cylinder

Following Fetter [10] we use a hydrodynamic model of a weakly damped, compressible charged electron fluid placed in a rigid, neutralizing positive background set to  $+en_0$ . The electron density is written as  $n_0 + n$ , where  $n$  is a small perturbation, and the electronic velocity field is denoted  $\mathbf{v}$ . Finally, we include the electrostatic potential  $\Phi$  and neglect radiation effects. The basic equations for the system are the linearized versions of the continuity equation and of Euler’s and Poisson’s equations [10]:

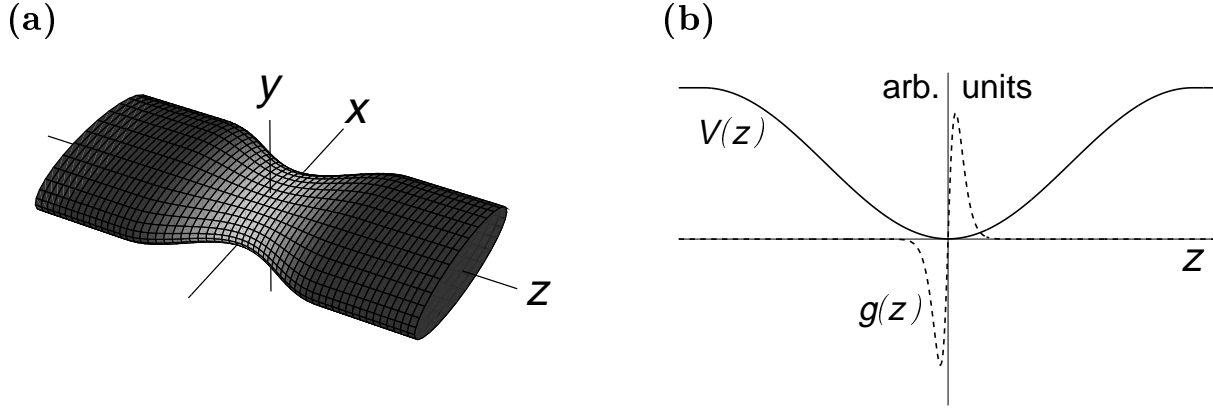
$$\partial_t n = -n_0 \nabla \cdot \mathbf{v} \quad (1)$$

$$\partial_t \mathbf{v} = -\frac{s^2}{n_0} \nabla n + \frac{e}{m} \nabla \Phi \quad (2)$$

$$\nabla^2 \Phi = \frac{en}{\epsilon}. \quad (3)$$

Here  $s = (\partial P / \partial n) / m = \sqrt{3/5} v_F$  is the sound velocity of the liquid. Combining equations (2) and (3) and introducing the plasma frequency  $\omega_p = e^2 n_0 / m \epsilon$  we obtain a wave equation for  $n$ :

$$-s^2 \nabla^2 n + \omega_p^2 n = \omega^2 n. \quad (4)$$



**Figure 1.** (a) The squeezed elliptical cylinder. High and low densities are represented by dark and light shading respectively. (b) The effective potential  $V(z)$  (full curve) is determined by the parameter values of [9]. For this potential a solution of the wave equation (6) for  $g(z)$  is found numerically (broken curve). The solution represents a localized plasmon with an energy of the order of 10 K.

For the case of a homogeneous electron liquid confined in a cylinder of radius  $R$  we let all fields have the dependence  $f(r, \theta, z, t) = f_l(r) \exp[i(l\theta + qz - \omega t)]$ , with  $f_l$  being a Bessel function. Outside the cylinder  $\Phi(r)$  must decay and fulfil equation (3) with  $n = 0$ , and so  $\Phi_>(r) \propto K_l(qr)$ . Inside the cylinder,  $\Phi_<(r)$  can be either decaying, as  $I_l(\kappa r)$ , or oscillating, as  $J_l(\kappa r)$ . The lowest lying modes are the decaying ones reminiscent of surface plasmons. The oscillation frequency  $\omega$  is found by enforcing the boundary conditions that  $\Phi(r)$  and its derivative are continuous at  $r = R$ , and that the normal component  $v_r$  of the velocity vanishes at the surface. The solution is

$$\omega^2 = qR I_1'(qR) \left( K_l(qR) - \frac{q}{\kappa} \frac{K_l'(qR)}{I_1'(\kappa R)} I_1'(\kappa R) \right) \omega_p^2 \xrightarrow{q \rightarrow 0} \frac{e^2 n^{1D}}{2\pi m \epsilon} q^2 \ln \left( \frac{2}{qR} \right) \quad (5)$$

where we also have given the 1D limit arising as  $q \rightarrow 0$ .

### 3. Plasmons of an inhomogeneous electron liquid in a squeezed elliptical cylinder

Next, to approach the point contact geometry we introduce two perturbations. First, the cylinder containing the inhomogeneous electron liquid is squeezed geometrically in a region of length  $2\Lambda$  around  $z = 0$ , i.e. the radius becomes a function of  $z$ , say for example  $R(z) = R_0 - \delta R [1 + \cos(\pi z/\Lambda)] \Theta(\Lambda - |z|)$ . Similarly, a static  $z$ -dependent dip is imposed on the positive background charge density  $n_0$  inside the squeezed cylinder, say  $n_0(z) = n_0 - \delta n [1 + \cos(\pi z/L)] \Theta(L - |z|)$ .

In the adiabatic limit where derivatives of  $R(z)$  and  $n_0(z)$  are neglected, the wave equation (4) remains separable in cylindrical coordinates, and we make the ansatz  $n(r, \theta, z) = J_l(\kappa r) g(z) \exp[i(l\theta - \omega t)]$ , where  $J_l$  is a Bessel function and  $g(z)$  an arbitrary function to be determined. The boundary condition  $v_r(R(z)) = 0$  translates into a Neumann boundary condition  $J_l'(\kappa R(z)) = 0$  and consequently the ‘wavenumber’  $\kappa$  becomes a function of  $z$ ,  $\kappa = \kappa_{nl}(z) = \tilde{\gamma}_{nl}/R(z)$ , with  $\tilde{\gamma}_{nl}$  being the  $n$ th root

of  $J_l'(x)$ . Furthermore,  $\omega_p^2$  also becomes a function of  $z$ , since  $\omega_p^2(z) = e^2 n_0(z)/m\epsilon$ , and similarly for the sound velocity,  $s = s(z) \propto n_0(z)^{1/3}$ . As a consequence the wave equation (4) for  $n$  is changed into the following eigenfunction equation for  $g(z)$ :

$$-s(z)^2 \partial_z^2 g(z) + [s(z)^2 \kappa_{nl}^2(z) + \omega_p^2(z)] g(z) = \omega^2 g(z). \quad (6)$$

This is equivalent to Schrödinger’s equation (with a position-dependent mass) as seen by the identifications  $s^2 \leftrightarrow \hbar^2/2m$  and  $s(z)^2 \kappa_{nl}^2(z) + \omega_p^2(z) \leftrightarrow V(z)$ . Since  $\omega_p^2(0) < \omega_p^2(\pm\infty)$  bound states, i.e. localized plasmons, may exist. The ‘effective potential’  $V(z)$  is a sum of two terms; one,  $\omega_p^2$ , is dipping down on the length scale  $L$ , the other,  $s^2 \kappa_{nl}^2$ , is peaking on the length scale  $\Lambda$ . Depending on the relative strengths, shapes and length scales of the two terms the effective potential will appear rather differently. However, for realistic parameters, where the density variation dominates, we conclude that localized plasmons may exist in the squeezed, inhomogeneous cylindrical electron liquid as shown in figure 1.

The previous considerations dealt with a cylindrical geometry, but it is not difficult to approach the 2D case. The trick is simply to use elliptical coordinates  $(u, v, z)$  defined by  $(x, y, z) = (\eta \cosh(u) \cos(v), \eta \sinh(u) \sin(v), z)$ . The parameter  $\eta$  relates to the eccentricity of the ellipse. With these coordinates the wave equation separates as before. Instead of trigonometric functions of the angle  $\theta$  we now obtain the Mathieu functions of the generalized angular variable  $v$ , and instead of Bessel functions we obtain the modified Mathieu functions of the generalized radial coordinate  $u$ . By letting the eccentricity  $\eta$  tend to infinity we end up with a 2D geometry close to the one realized in the quantum point contact experiments. The conclusions obtained for the circular cylinder can be restated for the elliptic cylinder, and thus localized plasmons are expected to exist in or near the constriction region of quantum point contacts.

#### 4. Plasmon damping

So far we have treated only the undamped case. In real systems the collective plasmons are damped through their interaction with individual electron–hole pairs, the so-called Landau damping. This effect could be simulated by adding a damping term  $-v/\tau$  to the right-hand side of the Euler equation (2). Instead we will leave the classical level of description and continue with a microscopic quantum treatment. The classical level is adequate for demonstrating the existence of the collective (almost classical) plasmon excitations, but fails when it comes to single-particle effects.

The point contact can be approximated by a 1D region, the constriction, connected at each end to 2D regions, the contacts. For this 2D–1D–2D model of the point contact we can estimate the frequency of the confined plasmon using our insight from the classical calculations: we calculate the dispersion relation for an infinite 1D-wire and insert the wavevector  $q_c = 2\pi/L$ ,  $L$  being the length of the constriction and hence related to the size of the localized plasmon. The long-wavelength limit of the dispersion relation found in the random phase approximation (RPA) is

$$\omega_L^{\text{1D}} = \sqrt{v_f^2 + \frac{\gamma e^2 n^{\text{1D}}}{4\pi \epsilon m^*}} q_c. \quad (7)$$

Note how the second term under the square root resembles the classical result of equation (5). In [9] we used this formula successfully to fit the measured activation temperatures mentioned in the introduction.

The confined 1D plasmons will be Landau damped through their coupling to the 2D contacts outside the constriction. Inserting  $q_c$  in the RPA expression for the polarizability  $\chi^{2\text{D}}$  we obtain the following rough estimate of the lifetime  $\tau_p^{-1}/\omega_L^{\text{1D}}$  of the 1D plasmon coupled to the 2D contacts of the 2D–1D–2D model:

$$\frac{\tau_p^{-1}}{\omega_L^{\text{1D}}} \approx V^{2\text{D}} \text{Im} \chi^{2\text{D}} \approx \frac{2\pi e_0^2}{q_c} \frac{m^* \omega_L^{\text{1D}}}{2\pi \hbar^2 v_F^{2\text{D}} q_c} \lesssim 1. \quad (8)$$

The plasmons are seen to be damped, but not over-damped.

#### 5. Quasiparticle lifetime

In the Landauer–Büttiker formalism the conductance is given by single-particle properties. Once a particle is launched in a given channel of the injecting lead the transmission probability amplitudes are governed by the elastic scattering matrix of the system. For a quasiparticle with a finite lifetime it is possible that a particle will decay before completing its traversal of the system. We propose that the observed deviation from perfect quantized

conductance is indeed due to the finite lifetime of the quasiparticles. Furthermore we suggest that the main contribution restricting the lifetime comes from scattering against the localized plasmons. As demonstrated above, the localized plasmons provide a well defined finite energy  $\hbar\omega_L$ . Through the Coulomb interaction the electrons will scatter against the plasmons and hence the quasiparticle lifetime and the transmission properties are affected. The resulting lifetime and additional resistance is expected to exhibit an activated behaviour,  $\tau^{-1} \propto \exp(-T_a/T)$ , since a finite energy is needed to excite the localized plasmon. We are thus led to identify the activation temperature with the energy of the localized plasmon:  $T_a = \hbar\omega_L/k_B$ .

#### 6. Conclusion

Using a hydrodynamic description of the electron fluid, we have shown that localized plasmons with a frequency  $\omega_L$  are likely to exist near the constriction of a point contact. We have sketched how a more complete microscopic quantum calculation may account for a quasiparticle lifetime broadening  $\tau^{-1}$  with a thermal activation behaviour  $\tau^{-1} \propto \exp(-\hbar\omega_L/k_B T)$ . We relate this broadening with conductance and are led to identify the recently measured activation temperature  $T_a$  for conductance deviations with the frequency  $\omega_L$  of the localized plasmon.

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