

Parametric Correlation of Coulomb Blockade Conductance Peaks in Chaotic Quantum Dots

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Abstract

We investigate the autocorrelator of conductance peak heights for quantum dots in the Coulomb blockade regime. Analytical and numerical results based on Random Matrix Theory are presented and compared to exact numerical calculations based on a simple dynamical model. We consider the case of preserved time-reversal symmetry, which is realized experimentally by varying the shape of the quantum dot in the absence of magnetic fields. Upon a proper rescaling, the correlator becomes independent of the details of the system and its form is solely determined by symmetry properties and the number of channels in the leads. The magnitude of the scaling parameter is estimated by a semiclassical approach.

1. Introduction

Since the first experiments dealing with coherence phenomena in microstructures in the 80's and the theoretical activity that followed, many experiments have probed several different universal features of mesoscopic devices. One important example of such systems are semiconductor quantum dots [1], the subject of our study. With the fast progress in nanotechnology, the experiments gained a lot in sophistication and were able to explore more subtle aspects of mesoscopic physics. One of the most interesting subjects in the field is related to the influence of chaos in the ballistic electronic transport [2–9].

Presently, we know that the statistical measures of conductance (distributions and correlators) in open systems are well described by universal curves obtained from Random Matrix Theory (RMT). However, the understanding of system-dependent properties (average conductance, correlation scales of external parameters, etc.) demands a complementary approach, which extracts relevant elements of the underlying classical dynamics or average quantum properties of the system at hand. In particular, the decay width of the conductance correlators as a function of either an external magnetic field or of variations in the shape of the quantum dot can be understood in semiclassical terms.

In this work we extend our recent analysis [9] of the correlator of Coulomb blockade conductance peak heights for quantum dots in the chaotic regime. Here we concentrate on the case of preserved time-reversal symmetry and generalize our previous semiclassical analysis. We use the same notation as in our previous paper.

The conductance G of mesoscopic devices, such as quantum dots, can be evaluated using the Landauer-

Büttiker formula [10, 11]. For quantum dots the latter becomes particularly simple and can be expressed in terms of the partial decay widths Γ_{cv} and partial decay amplitudes γ_{cv} [3]:

$$\Gamma_{cv} = |\gamma_{cv}|^2, \quad \text{with} \quad \gamma_{cv} = \sqrt{\frac{\hbar^2}{2m}} \int ds \chi_c^*(r) \psi_v(r), \quad (1)$$

where ψ_v is the eigenfunction of the resonance v inside the dot with the appropriate boundary condition and χ_c is the wave function at energy ε_v for the channel c belonging to one of the leads. The integration is performed over the contact region between the lead and the quantum dot. Equation (1) does not include barrier penetration factors [9], which should not be important for the case of one open channel in each lead – the case we analyze here and the one which is presently more relevant for comparison with experiments. In this framework, the influence of finite temperatures caused by the rounding of the Fermi distribution can be done straightforwardly. In the single-level limit $\Gamma_{cv} \ll kT < \Delta$ (Δ is the single-particle mean level spacing), the conductance peak height G_v corresponding to an *on resonance* measurement is given by [12]

$$G_v = \frac{2e^2}{h} \left(\frac{\pi}{2kT} \right) g_v, \quad \text{with} \quad g_v = \frac{\Gamma_{Lv} \Gamma_{Rv}}{\Gamma_{Lv} + \Gamma_{Rv}}, \quad (2)$$

where $\Gamma_{L(R)v}$ is the width of the resonance v corresponding to a decay into open channels in the $L(R)$ lead. In other words, $\Gamma_{L(R)v} = \sum_{c \in L(R)} \Gamma_{cv}$. If the quantum dot has any asymmetry (or if there is a small amount of disorder) and the leads are placed far from each other on the scale of the electron wavelength, Γ_{Rv} and Γ_{Lv} will not be identical and should, in fact, vary independently.

2. Parametric correlators and Random Matrix Theory

We are interested in universal properties of a system Hamiltonian H which depends on an external parameter X and whose underlying dynamics is chaotic. Without loss of generality, we shall illustrate our study by considering X as a measure of the shape deformation of the quantum dot boundaries (in Ref. [9] the parameter X was identified with the magnetic flux threading the quantum dot). Furthermore, we assume that in the absence of a magnetic field, H can be

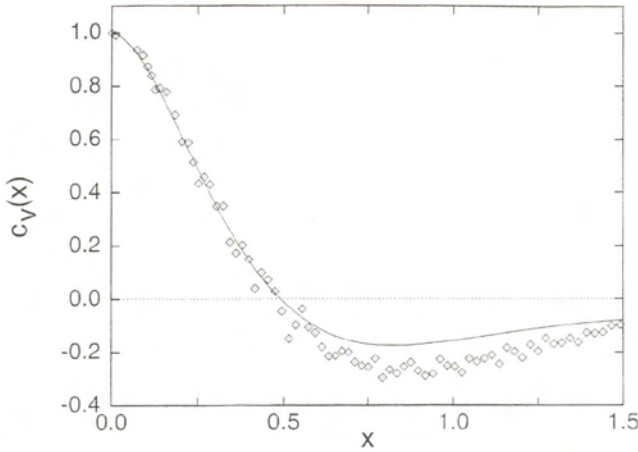


Fig. 1. The rescaled level velocity correlator $c_v(x)$ as a function of shape deformation. The full line is the RMT result for the GOE matrix diagonalizations while the points correspond to exact numerical calculations for the conformal billiard.

modeled as a member of the Gaussian orthogonal ensemble (GOE) of random matrices [13]. Thus, the statistical properties of the system should be obtained by an ensemble averaging. We focus our study on two autocorrelation functions: the level velocity correlator $C_v(X)$ and the conductance peak correlator $C_g(X)$.

As originally found by Szafer, Altshuler, and Simons [14, 15] the level velocity correlator

$$C_v(X) \equiv \frac{1}{\Delta^2} \left[\left\langle \frac{d\varepsilon_v(\bar{X} - X/2)}{d\bar{X}} \frac{d\varepsilon_v(\bar{X} + X/2)}{d\bar{X}} \right\rangle - \left\langle \frac{d\varepsilon_v(\bar{X})}{d\bar{X}} \right\rangle^2 \right], \quad (3)$$

is a universal function for systems whose underlying classical dynamics is chaotic. Here $\langle \dots \rangle$ denotes an average over resonances v and over different values of \bar{X} . Due to the ergodic nature of the systems under investigation, one usually conjectures that $\langle \dots \rangle$ is equivalent to an average over an ensemble of random Hamiltonians. The universality becomes explicit after applying the rescalings

$$x = X\sqrt{C_v(0)} \quad \text{and} \quad c_v(x) = C_v(X)/C_v(0). \quad (4)$$

It is very difficult to measure $C_v(X)$ experimentally for quantum dots and, consequently, to obtain the scale $C_v(0)$. The main goal of this work is to address this difficulty by suggesting an alternative way of studying parametric corre-

lations. We show in Section IV that one can estimate $C_v(0)$ by semiclassical arguments, once details of the confining geometry of the dot are known. An important statement implicit in Refs. [14] and [15] is that the quantity $\sqrt{C_v(0)}$ sets the scale for any averaged parametric function $\langle f(X) \rangle$. Therefore, we also study another (experimentally more accessible [23]) function, the conductance peak height correlator $C_g(X)$, which is defined as [8, 9]

$$C_g(X) \equiv \langle g_v(\bar{X} - X/2)g_v(\bar{X} + X/2) \rangle - \langle g_v(\bar{X}) \rangle^2. \quad (5)$$

This correlator also becomes universal upon rescaling X according to eq. (4) and $c_g(x) = C_g(X)/C_g(0)$. For the GOE case an asymptotic expansion of $c_g(x)$ for $x \ll 1$ can, in principle, be found analytically, in analogy to the GUE case shown in Ref. [9]. Here, however, we only present results based on numerical simulations, which nevertheless cover the entire X range. For that purpose, we have performed a series of exact diagonalizations of random matrices of the form $H(X) = H_1 \cos(X) + H_2 \sin(X)$, with H_1 and H_2 denoting two 500×500 matrices drawn from the GOE. This model and the details of the calculation are discussed in Ref. [9]. The final results for $c_v(x)$ and $c_g(x)$ are presented as full lines in Figs 1 and 2 respectively.

3. Dynamical Model

The aim of this section is to compare the statistical results of the previous section with exact numerical diagonalizations of a dynamical model. The essential characteristic of a dynamical model for this type of study is a fair resemblance with the actual experimental conditions, combined with its adequacy to numerical computations. For this reason we chose the (two-dimensional) conformal billiard [17, 5, 9]. Using complex coordinates, the shape of the billiard in the w plane is given by $|z| = 1$ in the area-preserving conformal mapping $w(z) = (z + bz^2 + ce^{i\delta}z^3)/\sqrt{1 + 2b^2 + 3c^2}$, where b , c , and δ are real parameters chosen in such manner that $|w'(z)| > 0$ for $|z| \leq 1$. In particular, we chose the values $b = c = 0.2$ and obtained different boundary shapes by varying δ . Hence, X is now to be identified with variations in δ . The classical dynamics of a particle bouncing inside billiards defined by these parameter values is fully chaotic [5]. The quantum dynamics is determined by choosing Neumann (zero current) boundary conditions, which yield non-trivial behavior at the boundary, as needed in eq. (1). The eigenstates ψ_v thus obtained correspond to the resonant wave function appearing in eq. (1) [5].

In practical calculations, one uses as a truncated basis the lowest (in our case 1000) eigenstates of the circular billiard ($b = c = 0$), which are known analytically. To solve the Schrödinger equation in this basis one has to calculate several thousand matrix elements of the Jacobian $\mathcal{J} = |w'(z)|^2$. However, using polar coordinate separation, only matrix elements of powers of the radial coordinate need to be calculated numerically. The angular part is done analytically and changes in shape are trivial, since b , c , and δ act as prefactors to the matrix elements. We have calculated the spectrum for $0.41 \leq \delta \leq 2.73$ in steps of 0.04, whereby the two values $\delta = 0, \pi$ which lead to spatial symmetry were avoided. Due to the truncation of the basis, only the lowest 250 of the calculated 1000 eigenstates were accurate enough to be used in the analysis. We also discarded the lowest 150

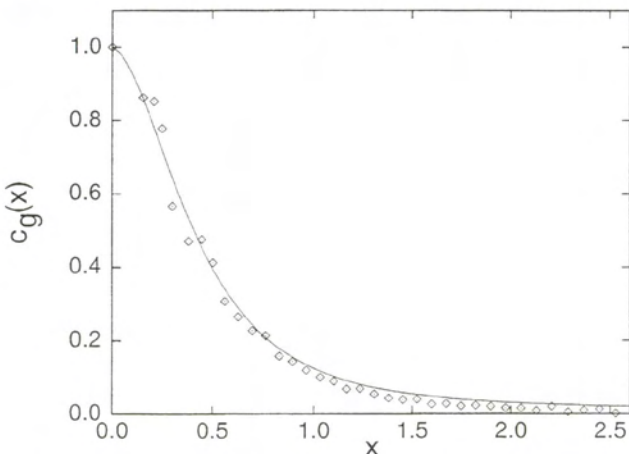


Fig. 2. The conductance peak height correlator $c_g(x)$. The symbols follow the same conventions of Fig. 1.

eigenstates because of their markedly nonuniversal behavior. The spectrum was then unfolded using the Weyl formula. We emphasize that it is not a trivial task to increase the number of converged states. No symmetry reduction of the resulting eigenvalue problem is possible for the asymmetric conformal billiard.

The scaling parameter $C_v(0)$ was calculated using eq. (3) and averaging over groups of 20 levels and 12 shapes. For magnetic flux variations we had found that $C_v(0)$ is proportional to $\varepsilon^{1/2}$. Here, for shape variations, we find the following fit:

$$C_v(0)_{\varepsilon, \delta} = a_\delta \varepsilon^{3/2}, \quad (6)$$

with $a_\delta = 0.0003 + 0.0014\delta$. A physical explanation for this energy dependence will be given in the next Section. We do not have, however, a full understanding of the δ -dependence of a_δ yet. Note that the numerical coefficients in a_δ are specific to a billiard with an area equal to π .

We found that spectral fluctuations were rather large and for that reason it seems more difficult to use shape than magnetic flux as the external parameter to obtain good statistics, at least in numerical simulations. Because of the δ -dependence of $C_v(0)$, the rescaling of X has to be performed with care. The linear relation $x = \sqrt{C_v(0)}X$ holds only locally. We therefore used the generalized relation

$$x(\varepsilon, X) \equiv \int_0^X \sqrt{C_v(0)_{\varepsilon, \delta}} d\delta. \quad (7)$$

In Figs 1 and 2 we show the numerical results for $c_v(x)$ and $c_g(x)$, respectively. They both compare rather well with the curves obtained from the GOE matrix diagonalizations.

4. Semiclassical estimates of $C_v(0)$

In a recent paper [16] Berry and Keating obtained an approximate expression for the level velocity correlator [eq. (3)] by combining derivatives of the cumulative level density expressed semiclassically with the Gutzwiller trace formula. Their calculation applied for the case where X is an Aharonov-Bohm flux line. In what follows we generalize the derivation presented in Ref. [16] for an arbitrary parametric variation of the Hamiltonian. Furthermore, here it will become more transparent why the result is universal for any chaotic system, depending only on whether the time-reversal symmetry is preserved or not. We do not consider the case of broken rotation symmetry (symplectic ensemble).

The derivation presented in Ref. [16] approximates the velocity level correlator as the following function of an external parameter X :

$$F_\eta(X, E) = \frac{1}{\delta\bar{X}} \int_{\delta\bar{X}} d\bar{X} \times \left\langle \frac{dN_\eta(E; \bar{X} - X/2)}{d\bar{X}} \frac{dN_\eta(E; \bar{X} + X/2)}{d\bar{X}} \right\rangle_{\delta E}, \quad (8)$$

where the average is taken over a window of width $\delta E \gg \Delta$ around E . One should view this approximation with a certain caution. It is in fact exact for $X = 0$ once the regularization parameter η is gauged properly. For large values of X , eq. (8) contains the leading order term of $C_v(X)$. For any other (intermediate) values of X , F_η just interpolates smoothly between the correct asymptotic values. The inaccuracy over intermediate values of X does not rely a priori

on the approximation used in the evaluation of F_η , but has a trivial origin: One cannot express $C_v(X)$ exactly by a 2-point, fixed energy correlator, as in eq. (8).

Equation (8) can be evaluated semiclassically by expressing the cumulative level density N_η by the trace formula $N_\eta(X; E) = \sum_\mu A_\mu \exp(iS_\mu/\hbar - \eta T_\mu/\hbar)$, where the sum runs

over all periodic orbits. The amplitude A_μ contains information about the stability of a certain orbit μ , with S_μ being its action, and T_μ its period. All A_μ , S_μ , and T_μ depend on E and X . The coefficient η acts as a regulator of the infinite summation. Below, our approach will cover only the range where the classical perturbation theory is valid, which, nonetheless, can correspond to large changes in the spectrum, depending on \hbar [18]. With that in mind, we assume that a shape distortion will only change the action of a periodic orbit by $S_\mu(X) \approx S_\mu(X_0) + Q_\mu(X - X_0)$, with $Q_\mu = dS_\mu(X_0)/dX$. The change in the orbit itself is assumed to be a higher order effect, allowing us to neglect variations in A_μ as compared to Q_μ/\hbar . After some standard manipulations, the diagonal term of F_η can be written as

$$F_\eta^{diag}(X; E) = \frac{2}{\hbar^2} \left\langle \sum_\mu |A_\mu|^2 Q_\mu^2 \exp\left(-\frac{i}{\hbar} Q_\mu X - \frac{2\eta}{\hbar} T_\mu\right) \right\rangle_{\delta E}. \quad (9)$$

Two further steps allow us to establish the connection between eq. (9) and the asymptotic part of level velocity correlator. Due to the chaotic nature of the system, the longer the orbits become, the more they explore ergodically the phase space. One can assign a scale T_{erg} to the time when periodic orbits with period $T > T_{erg}$ begin covering uniformly the phase space. For $T > T_{erg}$ the Hannay-Ozório de Almeida sum rule [19] is valid, and the sum in eq. (9) can be enormously simplified [20], since $\phi(T) = \langle \sum_\mu |A_\mu|^2 \delta(T - T_\mu) \rangle_{\delta T} \rightarrow 1/2\pi^2 |T|$. The average $\langle \dots \rangle$ is now taken over a small window δT which, nevertheless, contains a large number of periodic orbits. Notice that this expression specializes our results, since it is stated in a form suitable only for time-reversal symmetric systems. (It is trivial to extend our results to systems where this symmetry is absent.) In order to proceed analytically, one has to extrapolate the Hannay-Ozório de Almeida sum rule beyond its range of applicability, i.e., for $T < T_{erg}$ [21]. This approximation is not controllable, but its accuracy is supported by numerical evidence [9] Hence,

$$F_\eta^{diag}(X; E) \approx \int_0^\infty dT \phi(T) \left\langle Q_\mu^2 \exp\left(\frac{i}{\hbar} Q_\mu X\right) \right\rangle_{\delta T}, \quad (10)$$

with $\langle \dots \rangle_{\delta T}$ now indicating an average over periodic orbits within a window δT , but at a fixed energy E . This average can be easily handled following the ansatz proposed in Ref. [22] which reads

$$P(Q) = \frac{1}{\sqrt{2\pi \langle Q^2 \rangle_{\delta T}}} \exp\left(-\frac{Q^2}{2 \langle Q^2 \rangle_{\delta T}}\right),$$

with

$$\langle Q^2 \rangle_{\delta T} = \alpha(E)T \quad (11)$$

where $P(Q)$ is the probability density of Q . Thus, α is a system-dependent quantity which contains information about the long-time dynamics, therefore probing some of its

global (average) properties. For billiards in the semiclassical regime, the energy dependence of α follows straightforwardly from the fact that to a trajectory of length L corresponds an action $S = (2mE)^{1/2} L$, yielding

$$\begin{aligned} \langle Q^2(T) \rangle_{\delta T} &= 2mE \int_0^T dt' \int_0^T dt \left\langle \frac{d\dot{L}(t')}{dX} \frac{d\dot{L}(t)}{dX} \right\rangle_{\delta T} \\ &\approx 2mET \left\{ \int_0^\infty dt C(t) \right\}. \end{aligned} \quad (12)$$

This relation is similar to that obtained in Ref. [9] for the variance of the winding number of classical trajectories. Here, $d\dot{L}(t)$ gives the rate in time by which the length of a trajectory changes under a variation dX in the billiard shape. For a chaotic system one expects not only that $\langle d\dot{L}(t)/dX \rangle_{\delta T} = 0$ (because different trajectories are uncorrelated), but also that the autocorrelator function of $d\dot{L}(t)/dX$ decays sufficiently fast to make the integral over $C(t)$ finite. The magnitude of $\int dt C(t)$ is determined solely by the velocity of the particle and the billiard area \mathcal{A} . By a proper rescaling one can write $\alpha(E) = (8mE^3 \mathcal{A})^{1/2} \kappa$, where κ is a classical, dimensionless quantity computed for an unit area billiard for trajectories with unit velocity. The energy dependence of α is markedly different from the case where the magnetic field plays the role of the external parameter. Notice that $\alpha(E)$ matches the largest contribution to $C_\nu(0)$ found in our numerical fit [eq. (6)].

At this point the semiclassical interpretation of universality becomes apparent: The classical long-time dynamics of chaotic systems has some universal properties which are the only necessary elements to evaluate density-density correlators in the semiclassical approximation. In addition to that, from the classical dynamics one can obtain system-specific information which helps to understand relevant scales for different experimental situations.

After replacing the average of the term involving Q_μ by an integral over the distribution defined in eq. (11), we obtain

$$F_{\eta=\Delta/2\pi}^{diag}(0; E) = \frac{\alpha(E)}{\pi\hbar\Delta}, \quad (13)$$

Equation (13) gives a semiclassical estimate for $C_\nu(0)$. Upon the usual rescaling [eq. (4)], we get

$$f(x; E) = \frac{1 - \pi^2 x^2/2}{(1 + \pi^2 x^2/2)^2}, \quad (14)$$

which, as expected, exhibits the correct large- x asymptotic behavior for time-reversal symmetric systems [15], namely, $f(x; E) \rightarrow -2/(\pi^2 x^2)$ for $x \gg 1$.

The classical evaluation of κ makes it possible to predict $C_\nu(0)$. For the conformal billiard, for instance, this implies in a simulation of classical trajectories in slightly different geometries. We emphasize that in order to give an accurate prediction for κ in experiments, it is necessary to know very precisely the actual geometry of the quantum dot in consideration.

5. Conclusions

In this paper, we have argued that the universal form of the parametric correlator of conductance peak heights indicates

the chaotic nature of the electron dynamics in quantum dots in the Coulomb blockade regime. In the absence of magnetic fields, the simplest parameter to vary experimentally is the shape of the quantum dot. Whereas RMT provides the framework for obtaining the universal form of the correlation function, the nonuniversal correlation scale can be understood in simple semiclassical terms. This scale is rather sensitive to the geometry of the dot and the Fermi energy.

We have compared a theoretical curve obtained from numerical simulations of random matrices with the exact correlator obtained for the conformal billiard after averaging over energy and shape deformation. The agreement found was good, given the limitations imposed by the size of our data set. We also found that the semiclassical theory can explain the energy scaling of $C_\nu(0)$ on very generic grounds – shedding some light on how to make quantitative predictions about nonuniversal characteristics of such systems from a different (purely classical) perspective. We remark, however, that shape variations are harder to treat classically, since they alter trajectories strongly. To obtain fair estimates of the scaling one needs to evaluate $\langle Q^2 \rangle_{\delta T}$ with great numerical accuracy and use very small increments in shape deformation. Further investigation in this line is required to make a firm statement. It is interesting to note, though, that the energy dependence of the scaling is distinct for variations of boundary shape or external magnetic field.

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