ELSEVIER

Surface Science 305 (1994) 490–494

surface science

Universal fluctuation effects in chaotic quantum dots

A. Douglas Stone *, Henrik Bruus

Applied Physics, Yale University, P.O. Box 2157, New Haven, CT 06520, USA

(Received 19 April 1993; accepted for publication 19 May 1993)

Abstract

We show that quantum dots with classically chaotic dynamics exhibit universal quantum fluctuation effects in a number of physical properties. Specifically, the distribution of Coulomb blockade resonance amplitudes and the correlation function of the quantum dot energy levels with shape distortion have universal forms which are shown to agree well with numerical results from a billiard model of quantum dots. These universal statistical properties are experimentally accessible through transport and capacitance measurements of single dots; a clear experimental signature may be obtained by their sensitivity to a weak magnetic field. Integrable models for quantum dots are found to have non-universal statistical properties.

The analogy between semiconductor quantum dots and "artificial atoms" has been widely noted and has been the basis for various model calculations which include the effects of size quantization and Coulomb interactions. However, unlike natural atoms, the potential which confines the electrons within each quantum dot will vary from dot to dot due both to variations in the fabrication process leading to confinement and to the weak potential fluctuations arising from randomly located remote donors. Although these potential variations are small enough that treating these systems as short-range disordered (i.e. elastic mean free path smaller than sample size) is not realistic, they are likely to be large enough to generate chaotic classical motion for trapped electrons. It is now understood that there are strong similarities between the behavior of disordered quantum systems and simple quantum systems which are classically chaotic, and that therefore quantum-chaotic systems should exhibit many of the mesoscopic fluctuation effects characteristic of disordered system [1–5]. Quantum dots are of particular interest because they are *isolated* quantum systems with relatively large level-spacing $\Delta\epsilon$ (~ 0.05 meV or ~ 500 mK) and therefore can be studied in the regime $kT < \Delta\epsilon$ where one expects the maximal effect due to quantum-chaotic fluctuations.

In fact there are many experimental measurements on single quantum dots which indicate the presence of such fluctuations. Three examples are: (1) The amplitude of the Coulomb-blockade (CB) oscillations fluctuates non-monotonically by as much as an order of magnitude between adjacent peaks [6–8]. (2) The amplitude pattern is completely rearranged in an apparently random (but reproducible) fashion by magnetic fields of order 20 mT [8,3]. (3) Recent capacitance measurements of the spectrum of single dots show non-monotonic variations as a function of mag-

* Corresponding author.

0039-6028/94/$07.00
SSDI 0039-6028(93)E0596-M

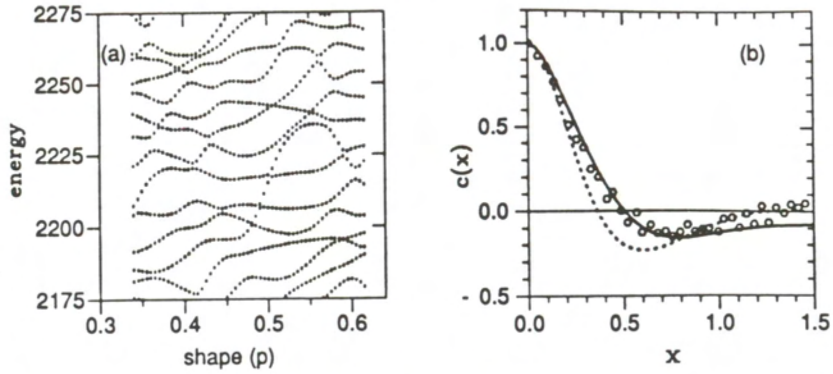

Fig. 1. Features of the 2d quantum dot with a hard-wall boundary given by $w(z) = \cos(p)z + \sin(p)/\sqrt{2}z^2$, $|z| = 1$. In (a) are shown the (even parity) energy levels 268 to 279 vs. shape parameter p. In (b) is shown the correlation function, $c(x)$, defined in Eq. (1). The full (broken) curve corresponds to the orthogonal (unitary) case. The open circles are our numerical results.

netic field not related to Landau level formation [9].

In earlier work, Jalabert et al. [2] used random-matrix theory to predict the distribution of CB amplitudes and tested their prediction by calculating transmission resonances through irregularly-shaped quantum dots. The model studied there had three disadvantages: First, the classical mechanics generated by the dot potential was essentially unknown, although undoubtedly chaotic. Second, no characterization of the spectrum in isolation was possible. And third, for technical reasons the time-reversal symmetry breaking transition predicted by the analytic the-

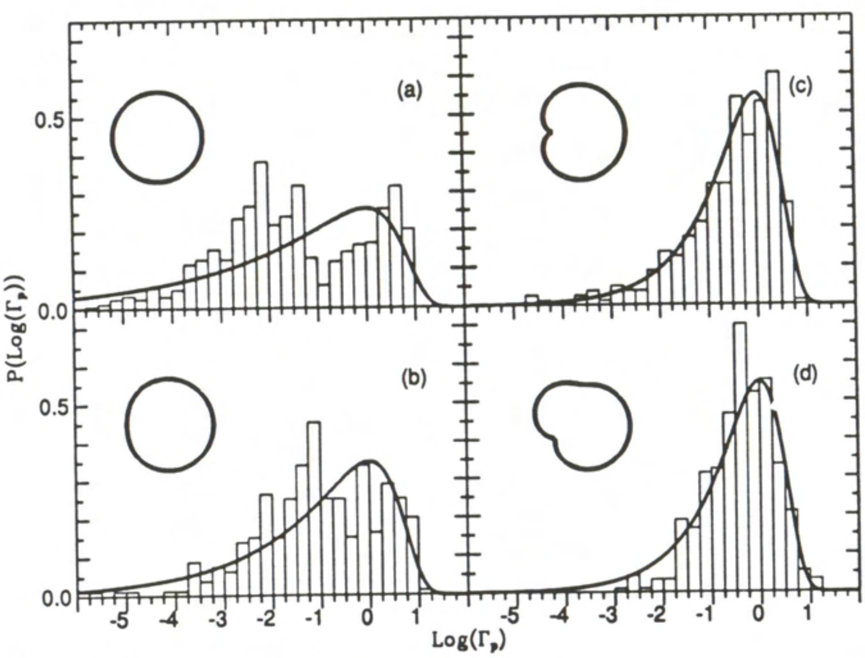

Fig. 2. The distribution of the partial decay width, Γ_p, for four different shapes of the quantum dot. Shown are histograms representing our numerical results, χ_ν^2 distributions (full curves) serving as a guide for the eye ($\nu = 0.33$, 0.50, 1.00, and 1.00, respectively), and as inserts the particular shapes of the dot. In (a), (b), and (c) we used the simple quadratic map with $p = 0.0$, 0.2, and 0.5, respectively. In (d) we used the cubic map with $\phi = \pi/3$, $b = c = 0.2\ a$, and $a = 0.913$. In (a) the model is integrable, in (b) it is nearly integrable, and in both (c) and (d) it is fully chaotic.

ory was difficult to test quantitatively. Here we study a quantum billiard model [12–14] which corrects all of these deficiencies (although we need to introduce one further assumption to connect our results to scattering theory [3]).

The model we study consists of all two-dimensional infinite potential wells with boundaries given by the cubic conformal mapping $w(z) = az + bz^2 + ce^{i\phi}z^3$ (where a, b, c are chosen to fix the area to the value π). Often it suffices to set $c = 0$ in which case $a = \cos(p)$, $b = \sin(p)/\sqrt{2}$ and p is the single parameter which varies the shape. The mapping is conformal for $0 \leq p \leq 0.615$. At $p = 0$ we have a circular well with integrable classical motion (angular momentum is conserved) and Bessel function eigenstates. For $p \approx 0.34$ the curvature at $z = -1$ vanishes and the well begins to develop a concave region which leads to fully chaotic classical dynamics [12]. The shape of the well for four different parameter choices is shown in Fig. 2 (insets). The basic simplification of the model is that the Schrödinger equation for the energy levels of irregular billiards with non-zero p-values may be solved by a very efficient numerical procedure which exploits the change of variables defined by the inverse conformal transformation [13].

In Fig. 1 we plot typical high energy levels as function of shape. Note the rapid pseudo-random oscillations for very small shape distortions. We can characterize this fluctuating spectrum by the correlation function of the derivatives of the energy levels with respect to p. Recent work [5,10] indicates that when properly rescaled this type of correlation function has a universal form in chaotic quantum systems. Defining $C(0) = \langle([\partial\epsilon/\partial p](1/\Delta\epsilon))^2\rangle$ and $x = \sqrt{C(0)}\,p$ the rescaled correlation function takes the form

$$c(x) = \left\langle \frac{\partial\epsilon_i(\bar{x}+x)}{\partial\bar{x}} \frac{\partial\epsilon_i(\bar{x})}{\partial\bar{x}} \frac{1}{\Delta\epsilon^2} \right\rangle. \quad (1)$$

We find good agreement with the behavior obtained from disordered quantum dots for the first few correlation lengths and then a substantial deviation as seen in other work [5,10]. It follows from the rescaling that the correlation "length" in shape is just $p_c \approx 1/\sqrt{C(0)}$. p_c decreases with increasing energy as expected due to the shortening of the electron wavelength. The strong dependence of the energy levels on shape has implications for the properties of quantum dot arrays. Since there is likely to be significant shape variation between dots one does not expect the formation of superlattice bands due to the random mismatch of energy levels on adjacent dots. In fact no such band-formation effects were observed in recent experiments [11] at zero or low magnetic field; it is only at fields $B > 2$ T that the mini-gaps appeared due to the formation of narrow Landau bands. Thus our results suggest that quantum dots are not in general well-described as identical "artificial atoms".

Not only will the spectrum of quantum dots fluctuate with shape (or magnetic field) but the probability density of the eigenstates of a given dot will fluctuate from level to level. We attribute the CB amplitude fluctuations to spatial fluctuations in the probability density of the quasi-bound states [2]. In the regime $\bar{\Gamma} < kT < \Delta\epsilon < e^2/C$, (where $\bar{\Gamma}$ is the mean level width and e^2/C is the charging energy) a single level controls each Coulomb blockade peak [15,16] and large amplitude fluctuations are observed. Due to thermal broadening the width of all CB peaks are $\sim kT$, whereas the amplitude [16] is given by

$$g_{\max} = \frac{e^2/h}{4\pi kT} \frac{\Gamma_\lambda^l \Gamma_\lambda^r}{(\Gamma_\lambda^l + \Gamma_\lambda^r)} = \frac{e^2}{4\pi h} \frac{\bar{\Gamma}}{kT} \alpha_\lambda, \quad (2)$$

where $\Gamma_\lambda = \Gamma_\lambda^l + \Gamma_\lambda^r$ is the total decay width for level λ and Γ_λ^l, Γ_λ^r are the partial decay widths into the right and left leads. The factor α_λ in Eq. (2) is a dimensionless measure of the *area* under the $T = 0$ resonance, hence the observed amplitude fluctuations reflect the fluctuations in these areas.

It is possible using the R-matrix theory developed for compound nuclear scattering [17] to express the level widths of Eq. (2) in terms of states $X_\lambda(x, y)$ of the dot in isolation (with appropriate boundary conditions); details are given elsewhere [4,3]. We assume that the resonance wavefunctions X_λ are described by the orthogonal random-matrix ensemble when time-reversal symmetry is present ($B = 0$) and by the unitary random-matrix ensemble when TR is broken (sufficiently high magnetic field). We note that this assumption should be reasonable even in the presence of strong interactions as long as one applies it to the self-consistent single-particle states, as is done, e.g., when analyzing shell model calculations of the nucleus. If X_λ are described by random-matrix theory then the distribution of partial widths Γ_p should have a χ^2_ν distribution with $\nu = 1, 2$ degrees of freedom [2]. This distribution should be universal in the chaotic regime, i.e. two different shapes both of which generate chaotic classical dynamics should have the same distribution of level widths (even though the individual levels are quite different (see Fig. 1)). However, if the system approaches integrability then non-universal distributions differing from χ^2 should arise. Precisely this behavior is confirmed by numerical calculations of the partial width distribution for the conformal billiard model as seen in Fig. 2, where only the fully chaotic models (c) and (d) fit a $\chi^2_{\nu=1}$ distribution reasonably well.

Having confirmed that random matrix theory works well in the chaotic regime, one can derive [2,4] from this ansatz the probability density $\mathscr{P}_\nu(\alpha)$, where $\nu = 2$ for the orthogonal case and $\nu = 4$ for the unitary case. One finds

$$\mathscr{P}_2(\alpha) = \sqrt{\frac{2}{\pi\alpha}}\ \mathrm{e}^{-2\alpha} \tag{3}$$

$$\mathscr{P}_4(\alpha) = 2\ \mathrm{e}^{-4\alpha}\int_0^\infty \mathrm{d}z\ \mathrm{e}^{-z}\sqrt{\frac{z+4\alpha}{z}}\ . \tag{4}$$

$\mathscr{P}_2$ and $\mathscr{P}_4$ are plotted in Fig. 3 where they are compared to numerical data obtained by evaluating α for the wavefunctions of the conformal model for the case $\nu = 2, 4$. The time-reversal symmetry-breaking needed to study $\nu = 4$ is achieved by adding an Aharonov–Bohm flux to the center of the quantum dot [14]. Note the substantial suppression of small peak amplitudes caused by breaking TR symmetry. This reduces substantially the variance of α, and from Eqs. (3) and (4) one finds $\Delta\alpha_4^2/\Delta\alpha_2^2 = 32/45 \approx 0.71$.

If this suppression of CB amplitude fluctuations due to TR symmetry breaking is to be observable the magnetic field necessary to induce the TR transition be small compared to that needed for Landau level formation. Landau level formation strongly suppresses the fluctuations [8]; the classical analogue of this effect is the suppression of chaos by the formation of stable skipping orbits. Adapting arguments by Berry and Robnik [14,3], we find that the flux, Φ_c, needed to break TR symmetry for a dot of area A is $\Phi_c \approx (\hbar v_f/\Delta\epsilon\sqrt{A})^{-1/2}h/e$. In the experimental systems of interest this corresponds to a field of order a few times 10 mT, much smaller than that needed for edge-state formation. Thus the statistical effect of time-reversal symmetry breaking predicted by our theory should be observable experimentally by making histograms of the amplitudes as a function of magnetic field. It is hoped that careful experimental tests of this type will be performed in the near future.

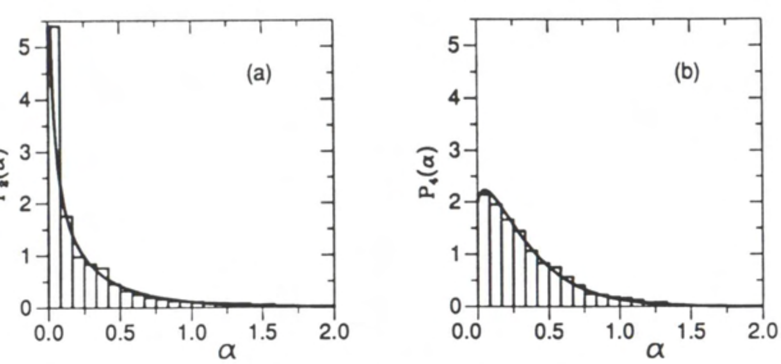

Fig. 3. Predicted distribution of peak amplitudes α in the presence of time-reversal symmetry (a) and absence (b), compared to the numerically generated amplitudes obtained from the cubic model in the fully chaotic case (same shape as in Fig. 2d).

A.D.S. gratefully acknowledges the contributions of his earlier collaborators R.A. Jalabert and Y. Alhassid. H.B. is supported by Grant No. 11-9454 from the Danish Natural Science Research Council. We want to thank B. Simons for helpful conversations and for sharing the numerical curves in Fig. 1b. This work was partially supported by ARO grant DAAH04-93-G-0009.

1. References

[1] R.A. Jalabert, H.U. Baranger and A.D. Stone, Phys. Rev. Lett. 65 (1990) 2442.
[2] R.A. Jalabert, A.D. Stone and Y. Alhassid, Phys. Rev. Lett. 68 (1992) 3468.
[3] A.D. Stone and H. Bruus, Physica B 189 (1993) 43.
[4] A.D. Stone, R.A. Jalabert and Y. Alhassid, in: Springer Series in Solid-State Sciences, Vol. 109, Eds. H. Fukuyama and T. Ando (Springer, Berlin, 1992) p. 39.
[5] A. Szafer and B. Altshuler, Phys. Rev. Lett. 70 (1993) 587.
[6] U. Meirav, M.A. Kastner and S.J. Wind, Phys. Rev. Lett. 65 (1990) 771.
[7] L.P. Kouwenhoven et al., Z. Phys. B 85 (1991) 367.
[8] E. Foxman, P. McEuen, M. Kastner, U. Meirav and S. Wind, unpublished.
[9] R. Ashoori et al., Physica B 189 (1993) 117.
[10] B. Simons and B.L. Altshuler, Phys. Rev. Lett. 70 (1993) 4063.
[11] L.P. Kouwenhoven et al., Phys. Rev. Lett. 65 (1990) 361.
[12] M. Robnik, J. Phys. A 16 (1983) 3971.
[13] M. Robnik, J. Phys. A 17 (1983) 1049.
[14] M.V. Berry and M. Robnik, J. Phys. A 19 (1984) 649.
[15] Y. Meir, N. Wingreen and P.A. Lee, Phys. Rev. Lett. 66 (1991) 3048.
[16] C.W.J. Beenakker, Phys. Rev. B 44 (1991) 1646.
[17] A.M. Lane and R.G. Thomas, Rev. Mod. Phys. 30 (1958) 257.