Hysteresis effects in mesoscopic rings in the quantum Hall regime

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Abstract. It is argued that the behavior of two-dimensional mesoscopic rings in the fractional quantum Hall regime is governed by a quantum number reminiscent of the fluxoid quantum number in superconducting cylinders. We propose a quantum interferometer-like experiment, which probe the role of this quantum number.

We consider a ring shaped two-dimensional sample with the inner radius R_1 and the outer radius R_2. In addition to the flux from the homogeneous magnetic field, B, we imagine that a Aharonov-Bohm flux (AB-flux) is provided by a thin solenoid threading the annulus. This arrangement makes it possible to change the flux through the annulus without changing the filling factor of the electron liquid. For simplicity we consider the situation, where the filling factor has the value $\nu = 1/m$, $m = 1, 3, 5, \ldots$. It is well known that there exist persistent edge currents in QHE samples. These are the currents that produce the orbital magnetic moment of the system. The persistence of these currents at integer filling factors can be understood, within a single particle description, as a consequence of the fact that there are no non-current carrying states, with a lower energy, into which the edge electrons can be scattered. In the following we shall argue that by a change of the AB-flux a circulating current with a considerable life time can be induced in the ring.

Imagine, initially, that all the electrons, except electron k, are kept at arbitrary fixed positions inside the ring. This is just a mathematical device to analyze the structure of the many body wave function [1]. Writing the orbital part of this wave function as $\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) = fe^{i\phi}$, the velocity field of electron k, $\boldsymbol{v}_k = \boldsymbol{S}_k/(\Psi\Psi^*)$, $\boldsymbol{S}_k$ being the probability current density, is then given as

$$\boldsymbol{v}_k = \frac{\hbar}{M}\nabla_k\phi + \frac{e}{M}\boldsymbol{A}_k, \tag{1}$$

where M is the effective electron mass, and $\boldsymbol{A}_k$ is the vector potential. It follows from (1) that

$$\oint \boldsymbol{v}_k \cdot d\boldsymbol{s}_k = \frac{\hbar}{M}\oint \nabla_k\phi \cdot d\boldsymbol{s}_k + \frac{e}{M}\Phi_R, \tag{2}$$

where the circulations are taken along a circle with radius R, $R_1 < R < R_2$, and where Φ_R is the flux enclosed by this circle. The circulation of the phase gradient can be written as

$$\oint \nabla_k\phi \cdot d\boldsymbol{s}_k = 2\pi(\sum_i p_i + S), \tag{3}$$

where $\sum_i p_i$ describes the contribution from particle-bound zeroes belonging to those electrons, which happen to have their positions fixed inside the circle of circulation, and where S describes the contribution from the "super-zero" constituted by the hole in the ring. If the filling factor is not exactly $1/m$, so that the liquid contains quasiparticles, $\sum_i p_i$ will also contain a contribution from the quasiparticle zeroes. Broadly speaking the super-zero contributes to the circulation of the phase gradient by an amount, which is equal to the contribution from the particle-bound zeroes that would have been present, if there had been no hole. Since the wave function must be single valued, it follows that the circulation of the phase gradient is an integral multiple of 2π. If we move some of the particles originally placed at fixed positions inside the circle of circulation to positions outside this circle, the previous statement still holds. But since S is unchanged, this implies that $\sum_i p_i$ must be an integer for any set of fixed electron positions. Consequently S must be an integer quantum number describing a global invariant of the given many body wave function. Releasing the fixed electrons and taking averages we obtain

$$\langle \oint \boldsymbol{v}_k \cdot d\boldsymbol{s}_k \rangle = 2\pi(\langle \sum_i p_i \rangle + S) + \frac{e}{M}\Phi_R. \tag{4}$$

Since $\langle \oint \boldsymbol{v}_k \cdot d\boldsymbol{s}_k \rangle$ obviously is independent of k it expresses the circulation of the hydrodynamic velocity field of the electron liquid.

Starting from a situation, where at the radial position R the velocity is zero, by the solenoid we adiabatically increase the encircled flux from Φ_R to $\Phi_R + \alpha\phi_0$. The quantum number S, being a global invariant, will remain unchanged. If also $\langle \sum_i p_i \rangle$ remained unchanged the current density at $R = \beta l$, where l is the magnetic length, would be given as $j = \frac{\alpha}{\beta}\frac{ne^2\phi_0}{M2\pi l}$. With the values $M = 0.065 m_e$, $n = 5 \cdot 10^{15} m^{-2}$, and $\nu = 1$, we get $j = (\alpha/\beta)253$ A/m. Experimentally critical current densities of the order 40 A/m has been observed [2]. In reality, as a response to the adiabatic increase of the flux, there will be an adjustment of $\langle \sum_i p_i \rangle$. However, incompressibility imposes limits to such an adjustment, essentially allowing the electron liquid only to be shifted as a whole [3]. Let us consider the case where $\alpha > 0$, so that the electron liquid will be shifted inwards. The strain thus imposed on the electron system implies that the inner boundary (the confinement potential) will stem the electron liquid causing an increase of the inner edge current, and a decrease of the outer edge current, so that a net circulating current is set up.

The ground state energy and its associated current density pattern are periodic functions of the solenoid flux, with the period ϕ_0. As the parameter α is increased beyond 1/2, the ground state is no longer the state with the original value of S, but another macroscopic wave function with the global label $S - 1$. However, due to its global nature S may not change easily. This then leads us to considerations of the stability of S against changes of the AB-flux. Let the AB-flux, $\alpha\phi_0$, be adiabatically increased. Eventually, at some critical value α_0, it will be energetically favorable for the electron system to yield to the strain by the formation of quasiparticles (quasiholes at the outer boundary and quasielectrons at the inner boundary). If the annulus is not extremely small α_0 can be much larger than one. For $\alpha > \alpha_0$ electronic charge can be transported from the inner to the outer boundary by quasiparticle mediated processes. These processes can change S and thereby relax the strain imposed on the electron system by the AB-flux. However, for a suitable geometry the relaxation time the of the circulating current may be quite long [4]. Thus we speculate that the activated resistance in a quantum interferometer-like setup [5] may display hysteresis. The resistance of the strained system will be larger than the resistance in the case where the system has become unstrained by internal relaxation processes or by thermal cycling.

In practice the solenoid arrangement may be rather difficult to realize. Instead the quasi-persistent current may be induced by a change of the external magnetic field. However, by changing the filling factor away from $\nu = 1/m$, quasiparticles are formed in the electron liquid and presumably pinned by disorder. Although this will somewhat reduce the strain, hysteresis may still be observed.

References

[1] B.I. Halperin *Helv. Phys. Acta* **56** 75 (1983)
[2] L. Bliek et al. *Semicond. Sci. Technol.* **1** 110 (1986)
[3] B.I. Halperin *Phys. Rev. B* **25** 2185 (1982)
[4] D.J. Thouless and Y. Gefen *Phys. Rev. Lett.* **66** 806 (1991)
[5] G. Timp et al. *Phys. Rev. B* **39** 6227 (1989)