A Precise Measurement of QHR at NIM

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Abstract—**In this paper equipment for precise measurement of the Quantized Hall Resistance (QHR) at NIM is described. The essential parts in this equipment are a resistance comparator of one-to-one ratio with a comparison uncertainty of 3 × 10^{-8} and two specially designed resistor networks for determination of the ratio between 12906.4035 Ω of QHR at $i = 2$ and 10 kΩ or 1 kΩ. The transfer procedure from QHR to 10 kΩ or 1 kΩ can be completed easily with this equipment by a few one-to-one comparisons with a total uncertainty of 5 × 10^{-8}.**

I. Introduction

The establishment of a resistance standard based on the quantum Hall Effect (QHE) is an outstanding achievement in fundamental metrology in recent years. In the recommendations of the 18th CCE Meeting, which were then adopted by the CIPM, the value R_{K-90} = 25812.807 Ω for the von Klitzing constant R_K, i.e., the Quantized Hall Resistance (QHR) at $i = 1$, was recommended to be adopted as a conventional value and used from January 1, 1990 worldwide to establish a laboratory reference standard of resistance [1]. NIM recognized the importance of this kind of resistance standard and started a program to build a QHR standard in 1987 with the cooperation and help of PTB and DFM. This effort was preliminarily successful and a SI value of the von Klitzing constant was obtained at NIM in 1988, which was submitted to the 18th CCE Meeting [2]. This measurement system was improved further and gave its contribution to the adjustment of the Chinese representation of the ohm in 1989. A resistance comparator of one-to-one ratio and two specially designed resistor networks for transferring the QHR value of 12906.4035 Ω ($i = 2$) to 10 kΩ or 1 kΩ were completed. These apparatuses and the experimental results are described below.

II. The Precise Comparator for One-to-One Ratio

This resistance comparator was designed for the comparison of one-to-one ratio, as shown in Fig. 1. It is an apparatus of the potentiometer type specially designed for the comparison of resistance in the 1–10 kΩ range. The resistors R1 and R2 to be compared are connected in series and the voltage drops across them are compared with a compensation voltage across R3, respectively, the difference voltages are read by a Keithley 181 digital nanovoltmeter. The use of this nanovoltmeter instead of the ordinary galvanometer presents some advantages. First, the $6\frac{1}{2}$ digit reading of the nanovoltmeter can be obtained automatically, thus the fine adjustment of the compensation voltage in the ordinary potentiometer is avoided. Secondly, the digital nanovoltmeter can feed the acquired data into a computer through the IEEE-488 bus. This is important in the practical measurement because the data to be processed are numerous. K1 is used to switch the potentiometer from the potential terminals of R1 to those of R2 or vice versa and K2 is used to reverse the current

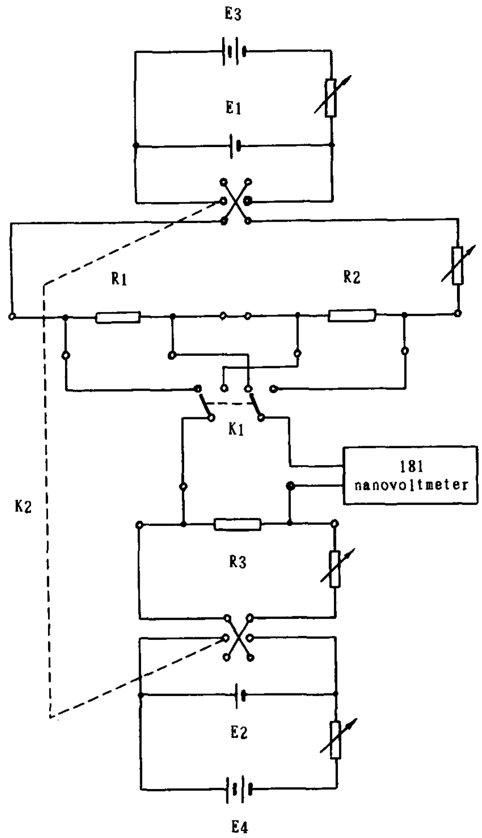

Fig. 1. The precise comparator for one-to-one ratio.

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direction in order to remove the effect of the thermal emf and other parasitic voltages. These two switches must be of very high quality because the resolution of the voltage measurement should be of some nanovolts. Commercially available programmable relay switches, e.g., scanner, etc., cannot be used, because these switches have rather high residual contact emfs. For example, the typical value for the emf of these switches is several hundred nanovolts. On the other hand, the heating of the relay coils is also a disturbance source. Therefore, two switches with low thermal emf, specially manufactured for the potentiometer, were used in the circuit shown in Fig. 1. The contact of these switches is made by a special kind of copper plate covered by a thick layer of silver and the contact pressure is quite low, thus the switches can give very good contact and have a long lifetime. The experiment showed that the variation of the contact emf of these switches is only several nanovolts and the contact resistance is below 1 mΩ, so that a reproducibility of several parts in 10^8 for the comparison of the QHR with 10-kΩ standards can be obtained. A difficulty with these switches is that they are operated manually by ordinary knobs and cannot be controlled easily by a computer. Therefore, a special driving mechanism was designed. As shown in Fig. 2, a permanent bar magnet is fixed at the knob of the switch. Above this magnet, an electromagnet produces a series of pulse forces to drive the switch according to the commands from the computer. The permanent magnet can keep its position without driving force. Therefore, the electromagnet only needs a short pulse current for driving the switch and the self-supporting current in the ordinary relays is unnecessary, thus the interference from both the heating of the electromagnet coil and the leakage of the power supply from the coil to the precise measurement circuit is avoided. At the same time, operation of the whole measurement circuit, data acquisition, and further processing can be completed automatically by a computer.

Great attention was paid to the insulation of the measurement circuit. The insulator and the supporting board for the switches and other parts are made of teflon material. Thus an insulation better than 10^{12} Ω was obtained to ensure the measurement uncertainty to be of the order of 10^{-8}.

Another difficulty for the apparatus in Fig. 1 is the relative drift of the current in the two independent circuits, i.e., the main circuit, including the standard and measured resistor, and the compensating circuit. The mercury batteries E1 and E2 were used as the current source because of their very low noise. For a comparison with an accuracy of a few parts in 10^8, it is necessary to keep the relative drift of the current in the two circuits within a limit of 1 part in 10^6 per hour and to use some suitable data acquisition program to remove the effect from the residual drift. But it has been found that it is very difficult to keep the relative drift within such a low limit. Therefore, a drift compensation was designed as shown in Fig. 1. The key point of this compensation is to inject an additional current into the battery E1 (or E2) from another battery E3 (or E4) with higher emf. This additional current is adjustable to ensure the total current from battery E1 (or E2) is close to zero, thus the output voltage of E1 (or E2) will be more stable. In addition, a strategy can be used to decrease the relative drift further, i.e., to create a very small drift by adjusting slightly the additional injected current. Thus the total residual drift from various reasons, e.g., the residual charge or discharge of battery E1 (or E2), the temperature variation, etc., can be compensated further and a very small relative drift below 1 part in 10^6 per hour can be obtained.

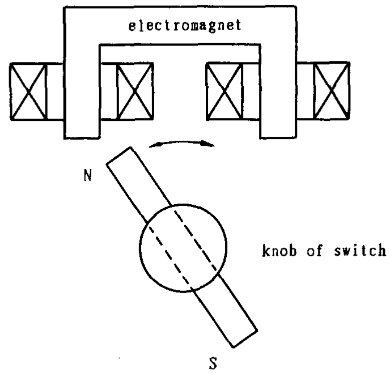

Fig. 2. The driving mechanism to operate the switches.

III. The Resistor Networks

The resistance reproduced by the QHR ($i = 2$) is 12906.4035 Ω. This value should be related with the legal Chinese representation of the ohm, therefore, two special resistance networks were designed and constructed for this purpose.

The first one is shown in Fig. 3. The function of this network is to relate two resistances with nominal values of 12906.4035 Ω and 10 kΩ or 1 kΩ. It consists of two resistors of 10 kΩ and three resistors of 1 kΩ. The first 10-kΩ resistor and three 1-kΩ resistors are connected in series. The second 10-kΩ resistor is isolated and can be connected to one of the 1-kΩ resistors in parallel by two links. It is easily shown that the whole resistance of such a network is

$$R = 10\ \text{k}\Omega + 1\ \text{k}\Omega + 1\ \text{k}\Omega + 1\ \text{k}\Omega \| 10\ \text{k}\Omega = 12909.0909\ \Omega. \tag{1}$$

It can be noticed that this value is very close to the value of the QHR ($i = 2$), i.e., 12906.4035 Ω. The relative difference between them is only about two parts in 10^4 and is easily compensated.

Every resistor in this network can be compared on the apparatus shown in Fig. 1. For example, if two links were removed, the second 10 kΩ resistor is isolated and can be compared with the first 10-kΩ resistor. The other three 1 kΩ resistors can also be compared with each other without difficulty. Thereby, if the ratio between 10 kΩ and 1 kΩ

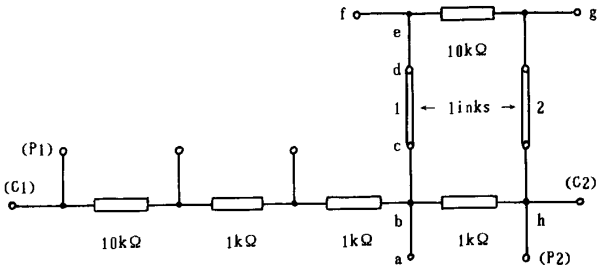

Fig. 3. The resistor network to determine the ratio of 12906.4035 Ω to 10 kΩ (or to 1 kΩ).

is known, the necessary ratio of 12909.0909 Ω (or 12906.4035 Ω) to 10 kΩ (or to 1 kΩ) can be derived precisely from a series of one-to-one ratio values between the resistances in the network.

The connection of this network was carefully considered. Thus every resistor in it can be compared with four terminal measurements by the apparatus shown in Fig. 1. At every junction with four branches, such as junction b and h, an equipotential structure mentioned in [3] was adopted to avoid the additional error due to the fourth extra branch. At the same time, it is very important to consider the error caused by links 1 and 2, because these links are connected in series with the second 10-kΩ resistor according to the configuration in Fig. 3. Therefore, the residual resistance of these links, including the resistance of the potential leads of the second 10-kΩ and the last 1-kΩ resistor, is added to the resistance of the second 10-kΩ resistor. But according to our design, these residual resistances can be measured in situ with four terminals. For example, if link 2 is removed and a current is injected into the terminal a and drawn out from f at the same time that g and (P2) act as the potential terminals, then the residual resistance between b and e, including the resistance of link 1, potential leads bc and de and the contact resistance at point c and d can be measured precisely as a total resistance. This strategy can also be used for link 2. Therefore, the effect from these two links can be corrected exactly.

The second network is a Hamon device, consisting of three resistors of 3 kΩ and one resistor of 1 kΩ, to obtain the ratio of 10 kΩ to 1 kΩ. The difference among three 3-kΩ resistors was controlled within a limit of one part in 10^5. At the same time, the resistance of both the potential and current leads of this Hamon device were adjusted to proper values. If the deviation of the resistance to the proper value is α_i for the potential leads and β_j for the current leads, the error due to the leads mismatch is proportional to the product $\alpha_i \beta_j$ and can be controlled within a limit of several parts in 10^9 [4]. Thus a ratio between 10 kΩ and 1 kΩ with an uncertainty below one part in 10^8 can be obtained by this Hamon device through several one-to-one comparisons.

As described here, the necessary ratio of 12906.4035 Ω to 10 kΩ can be simply determined by means of a series of one-to-one comparisons using the apparatus in Fig. 1 and two special resistor networks mentioned above. The uncertainty of this determination is about 3 parts in 10^8. For the whole procedure from QHR ($i = 2$) to 10 kΩ or 1 kΩ, the total uncertainty is estimated to be below 5 parts in 10^8.

IV. The Heterostructure Samples

Two kinds of GaAs/GaAlAs heterostructure sample were used in the experiment. The first kind of sample was manufactured at PTB and the Technical University of Braunschweig (Germany). The carrier concentration of the sample is about 4×10^{15} m^{-2} and the mobility is about 15–20 T^{-1} at 4.2 K. Two pairs of Hall probes are arranged for each sample. The contact was formed by alloying and diffusing tin balls directly on the sample surface, therefore, the photoetching technique is unnecessary. The second kind of samples were made at DFM and the H. C. Ørsted Institute (Denmark) by the photoetching technique and three pairs of Hall probes were formed. The contact is of the AuGeNi type. The carrier concentration and mobility at 4.2 K are in the same range as those of the German samples.

V. The Experimental Result

A series of experiments were completed during May 8–13, 1989. The comparison of different samples was satisfactory. The difference between QHR values ($i = 2$) reproduced by the two German samples is only 1×10^{-8}. The same difference between a German sample and a Danish one is also about 1×10^{-8}. Therefore, there is a strong evidence that the influence of the parallel conductance and other parasitic factors for these samples does not exceed a few parts in 10^8. The QHR resistance standard was also compared with the Chinese representation of the ohme Ω_{NIM}. This kind of comparison includes three steps. The first step is to compare the QHR at $i = 2$ with a resistor with the nominal value of 12906.4035 Ω. The uncertainty of this step is about 3×10^{-8}. The second step is to compare the resistor of 12906.4035 Ω with a resistor with the nominal value of 10 kΩ by means of the two special resistance networks mentioned above with an uncertainty of about 3×10^{-8}. The third step is to compare the 10 kΩ resistor with the Chinese representation of the ohm, i.e., the maintained 1 Ω national standard. This comparison was completed by means of two ordinary Hamon devices. One of them consists of ten 1 kΩ resistors and gives the ratio of the 10 kΩ to the 100 Ω. The other one consists of ten 10 Ω resistors and gives the ratio of the 100 Ω to the 1 Ω. The uncertainty for this third step is 1×10^{-7}. This rather large uncertainty is mainly from the unsatisfactory stability of the two Hamon devices used for the ratio of 10 kΩ to 1 Ω. Therefore, some improved Hamon devices are under construction.

The experiments during May 8–13, 1989 showed that the relation between R_K and Ω_{NIM} was

$$R_K = 25812.807 \times [1 + (1.470 \pm 0.109) \times 10^{-6}]\Omega_{NIM} \text{ (May 11, 1989).} \qquad (2)$$

The total uncertainty of (2) is estimated to be about 1.09 $\times 10^{-7}$.

From (2), the relation between R_K and $\Omega_{69\text{-}BI}$, the representation of the ohm maintained at the BIPM before January 1, 1990, can also be found. Considering the results of several recent international comparisons of resistance standards, the relative drift rate of Ω_{NIM} against $\Omega_{69\text{-}BI}$ can be estimated to be $-(0.0192 \pm 0.02)$ $\mu\Omega$ per year. According to [5], the time drift rate of $\Omega_{69\text{-}BI}$ against the SI resistance unit is $-(0.0614 \pm 0.0011)$ $\mu\Omega$ per year, thus the time drift of Ω_{NIM} should be $-(0.0806 \pm 0.02)$ $\mu\Omega$ per year. Then (2) becomes

$$R_K = 25812.807 \times [1 + (1.343 \pm 0.110) \times 10^{-6}]\Omega_{NIM} \text{ (Oct. 20, 1987)} \tag{3}$$

According to [5], on October 20, 1987,

$$\Omega_{NIM} = \Omega_{69\text{-}BI} + (0.435 \pm 0.037)\mu\Omega. \tag{4}$$

Therefore, it can be obtained that

$$R_K = 25812.807 \times [\,1 + (1.778 \pm 0.111) \times 10^{-6}]\Omega_{69\text{-}BI} \text{ (Oct. 20, 1987)}. \tag{5}$$

The measurements made at BIPM [5] gave

$$\begin{aligned} R_K &= 25812.8 \times [1 + (2.059 \pm 0.021) \times 10^{-6}] \cdot \Omega_{69\text{-}BI} \text{ (Oct. 20, 1987)} \\ &= 25812.807 \times [1 + (1.788 \pm 0.021) \times 10^{-6}] \cdot \Omega_{69\text{-}BI} \text{ (Oct. 20, 1987)}. \end{aligned} \tag{6}$$

It can be noticed that the results in (5) and (6) are in good agreement within the error limit.

The value given in (2) took an important part in the adjustment of the Chinese representation of the ohm according to the recommendation of 18th CCE Meeting. A representation of the ohm based on $R_{K\text{-}90}$ has been adoped in China since January 1, 1990.

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