Plateau Formation by Force from Pinning Centres in the Fractional Quantum Hall Effect

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1. Introduction

Consider a QHE sample containing N electrons. As the flux is increased (decreased) relative to the mid-plateau value, $mN\Phi_0$, by a given number of flux quanta, the electron liquid responds by forming the same number V_v (V_a) of fractionally charged vortices (antivortices) [1,2,3]:

$$V_v = \frac{\Phi}{\Phi_0} - mN \qquad \text{for } \Phi > mN\Phi_0$$
$$V_a = mN - \frac{\Phi}{\Phi_0} \qquad \text{for } \Phi < mN\Phi_0 \quad . \tag{1}$$

The vortex (antivortex) states arise because hereby the system is able, in a maximal way, to preserve the favourable $\nu = \frac{1}{m}$-correlations. Let us first consider a vortex state. In each vortex region there is a deficit of $\frac{1}{m}$ of an electron charge [4]. The deficit charge is pushed out into the area between the vortices and it is precisely because of this feature that the local filling factor, ν, in this area can be maintained at the value $\frac{1}{m}$ [5]. In an antivortex state charge is removed from the area between the antivortices causing that $\nu = \frac{1}{m}$ in this area and giving rise to an excess of $\frac{1}{m}$ of an electron charge within each antivortex region.

The vortices (antivortices) are assumed to be pinned. Considering a pinned vortex with a transport current flowing by,we show that the pinning centre exerts the force $\vec{f}_P = \vec{\Phi}_0 \times \vec{j}$ on the electron liquid, where $\vec{j}$ is the two-dimensional current density, $|\vec{\Phi}_0| = \frac{h}{e}$, and where $\vec{\Phi}_0$ is parallel to the magnetic field. For an antivortex the sign is opposite, $\vec{f}_P = -\vec{\Phi}_0 \times \vec{j}$. By inclusion of the force from the pinning centres in a force balance equation for the electron liquid, we show that the Hall response, V_H, is independent of the magnetic field in an interval around $B_{\nu=\frac{1}{m}}$. The physical nature of $\vec{f}_P$ is discussed.

2. The Force from a Pinning Centre

We consider the situation where the magnetic field is larger that the mid-plateau field, but where the density of vortices is not so large that they perturb each other. Now consider one particular of these vortices. This vortex, we imagine, is pinned, but to a "pin" that we can move around. Initially the "pin" is located at some position a. Let $\vec{f}_P$ denote the force which the pinning centre exerts on the electron liquid flowing by. It is this force we must supply when, quasi-statically (carefully avoiding other vortices) we move the "pin", and thus the surrounding vortex, from a to another position b. In doing this we perform the work

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$$W = \int_a^b \vec{f}_P \cdot d\vec{s} \quad . \tag{2}$$

This work must be equal to the increase of energy. However, since the overall result of the quasi-static transport is the displacement of the charge $-\frac{e}{m}$ from b to a (equivalent to the displacement of the positively charged vortex from a to b), we have

$$W = -\frac{e}{m} \int_a^b \vec{E} \cdot d\vec{s} \quad . \tag{3}$$

Since (2) and (3) are valid for arbitrary a and b we conclude that

$$\vec{f}_P = -\frac{e}{m} \vec{E} \quad . \tag{4}$$

However, the electrical field, $\vec{E}$, can be related to the current density that would exist at the position of the vortex if the vortex was not there. Noting that in the region between vortices the filling factor is maintained at its canonical value $\nu = \frac{1}{m}$ [5], we conclude that the current density is given by the quantum Hall relation

$$j = \frac{1}{m} \frac{e^2}{h} |\vec{E}| \quad . \tag{5}$$

Using the geometry of ref. [2], equation (5) implies $-\frac{e}{m}\vec{E} = \vec{\Phi}_0 \times \vec{j}$, and consequently the validity of

$$\vec{f}_P = \vec{\Phi}_0 \times \vec{j} \quad . \tag{6}$$

The case where the density of vortices is so large that the individual vortex to a considerable extent experiences the velocity field of other vortices is quite complicated, and we shall not discuss it here.

Let w denote the width of the sample, it then follows from (1) and (6) that the pinning centres exert the force

$$F_P = (\frac{\Phi}{\Phi_0} - mN)\Phi_0 \frac{I}{w} \tag{7}$$

on the electron liquid.

3. Plateau Formation

Since, in the QHE-experimental set-up, the transverse current is zero, the total transverse force on the electron system must be zero:

$$0 = F_M + F_E + F_P \ , \tag{8}$$

where $F_M = -\Phi\frac{I}{w}$ is the Lorentz force, and where $F_E = +eN\frac{V_H}{w}$ is the force from the Hall field.

From (7) and (8) we obtain $\Phi\frac{I}{w} = Ne\frac{V_H}{w} + (\frac{\Phi}{\Phi_0} - mN)\Phi_0\frac{I}{w}$ which implies

$$V_H = m\frac{\Phi_0}{e} I = m\frac{h}{e^2} I \quad . \tag{9}$$

Thus V_H comes out independent of the magnetic flux, i.e. as a Hall plateau.

4. The Physical Nature of the Pinning Force

Let us compare the two states of the electron liquid shown in fig. 1: the mid-plateau state which is without vorticity, and a vortex state containing one pinned vortex.

The velocity field and the density deviation of a vortex decay essentially exponentially [2,3]. Therefore, we assume, we can delimit a vortex region (the area within the dashed square in fig. 1(b)), outside which the flow of the liquid is unperturbed by the vortex. The linear extension of this vortex region is large compared to l , but small compared to the dimensions of the sample. The corresponding region, i.e. a region with the same location and shape, is displayed in fig. 1(a). In the mid-plateau state, fig. 1(a), the force balance for the electron liquid within the dashed region implies

$$\vec{f}_E^a + \vec{f}_M^a = 0 \quad , \tag{10}$$

where $\vec{f}_E^a$ is the electric force and $\vec{f}_M^a$ is the magnetic force. The force balance equation for the vortex region i.e. the dashed region i fig. 1(b) takes the form

$$\vec{f}_E^b + \vec{f}_M^b + \vec{f}_P = 0 \quad . \tag{11}$$

Since the average number of electrons in the vortex region is $\frac{1}{m}$ less than the average number of electrons within the dashed region in fig. 1(a) we have $\vec{f}_E^b = \vec{f}_E^a + \frac{1}{m}e\vec{E}$ which together with (4), (10) and (11) implies that

$$\vec{f}_M^a = \vec{f}_M^b \quad . \tag{12}$$

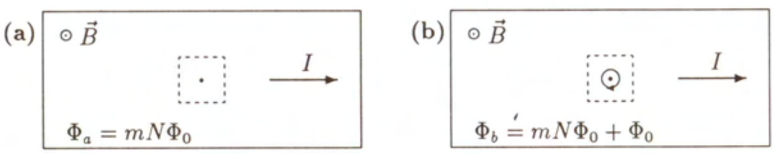

Figure 1: *The N-electron liquid at two different magnetic fluxes, Φ_a and Φ_b, in a sample with a pinning centre and a transport current I: the mid-plateau state (a) without vorticity ($\Phi_a = mN\Phi_0$) and a vortex state (b) containing one pinned vortex ($\Phi_b = mN\Phi_0 + \Phi_0$).*

Let A denote the total area of the sample. Noting that $B^b = B^a + \frac{\Phi_0}{A} \simeq B^a$ we conclude from (12) that the currents through the dashed regions in (a) and (b) are the same (a similar argument can be applied to the transition from the vortex state containing V_v vortices to a state containing $V_v + 1$ vortices). Although surprising, since the electron density is reduced in the vortex region, this conclusion can qualitatively be understood by inspection of fig. 2b.

As displayed in fig. 2c the magnetic force on the liquid in the vortex region is larger than the electric force. The physical significance of the pinning force, $\vec{f}_P$, is thus to balance the excess magnetic force.

Finally consider the case where the pinned vortex in fig. 1(b) is replaced by a pinned antivortex, the pinning centre now being a positively charged impurity. Again, by the

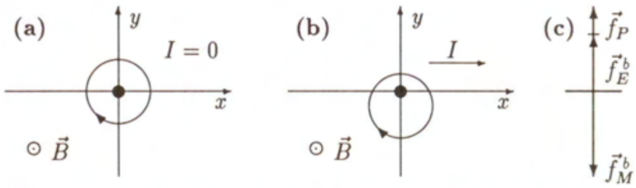

Figure 2: *In the currentless state we have rotational symmetry (a) of the density and the velocity field around the pinning centre. The passage of a transport current breaks this symmetry, and causes a displacement of the centre of the vortex relative to the pinning centre (b). Therefore the pinning centre (say a negatively charged impurity) repels the electron liquid above more strongly than below, thereby giving rise to a large current density in the upper vortex region. The forces on the electron liquid within the vortex region are shown in (c).*

same line of reasoning, we conclude that the currents through the two dashed regions are the same. However, in the antivortex case the pinning force balances an excess electric force.

5. References

1. S.M. Girvin: chap. 10 in *The Quantum Hall Effect* ed. by R.E.Prange and S.M. Girvin (Springer, Berlin, Heidelberg 1987)
2. H. Bruus, O.P. Hansen, and E.B. Hansen: Jour. Phys. C **21** L375 (1988)
3. H. Bruus, O.P. Hansen, and E.B. Hansen: to be published in Zeitschrift f. Physik
4. R.B. Laughlin: Phys. Rev. Lett. **50** 1395 (1983)
5. H. Bruus, O.P. Hansen, and E.B. Hansen: Proceedings of the 19. Int. Conf. on the Physics of Semiconductors, Warsaw August 1988